# Linear Algebraic Equations and Matrices

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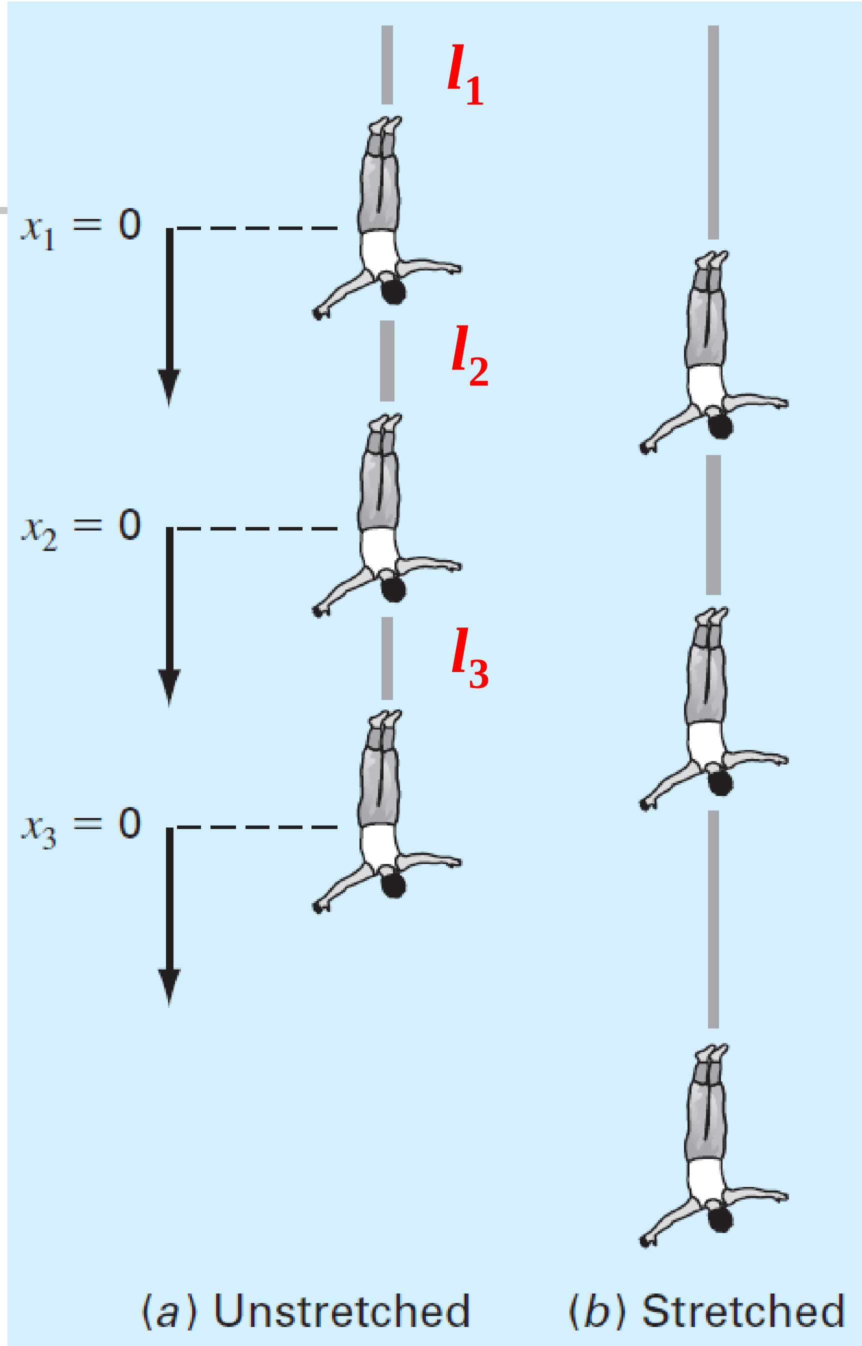
# Bungees jumpers problem

- 3 jumpers connected by bungee cords
- Determine the equilibrium positions of jumpers



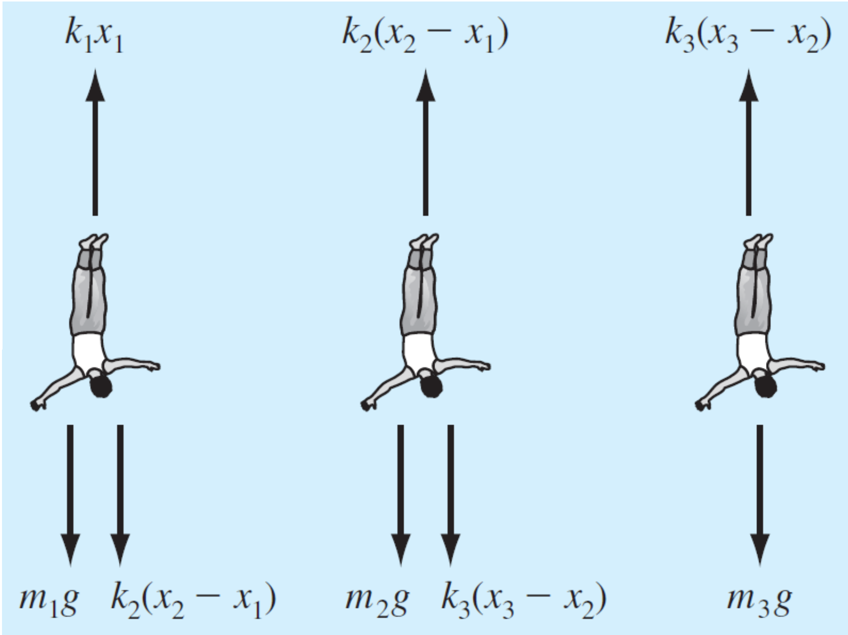
# Bungees jumpers problem (cont.)

- Assume that each cord behaves as a linear spring and follows Hooke's law

$$m_1 \frac{d^2 x_1}{dt^2} = m_1 g + k_2(x_2 - x_1) - k_1 x_1$$

$$m_2 \frac{d^2 x_2}{dt^2} = m_2 g + k_3(x_3 - x_2) + k_2(x_1 - x_2)$$

$$m_3 \frac{d^2 x_3}{dt^2} = m_3 g + k_3(x_2 - x_3)$$




steady-state solution, $d^2x/dt^2=0$

$$\begin{aligned}(k_1 + k_2)x_1 \quad - k_2 x_2 \quad &= m_1 g \\ -k_2 x_1 + (k_2 + k_3)x_2 - k_3 x_3 &= m_2 g \\ -k_3 x_2 + k_3 x_3 &= m_3 g\end{aligned}$$

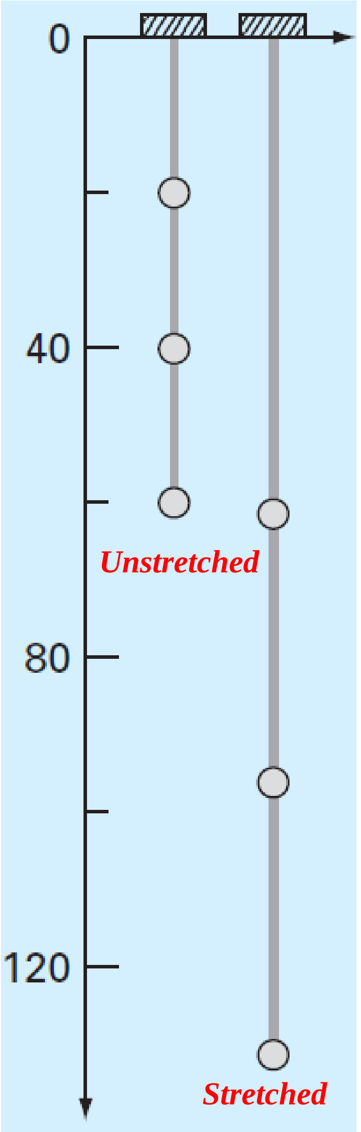
# Solving bungee jumpers problem with MATLAB

| Jumper | Mass (kg) | Spring Constant (N/m) | Unstretched Cord Length (m) |
|---|---|---|---|
| Top (1) | 60 | 50 | 20 |
| Middle (2) | 70 | 100 | 20 |
| Bottom (3) | 80 | 50 | 20 |

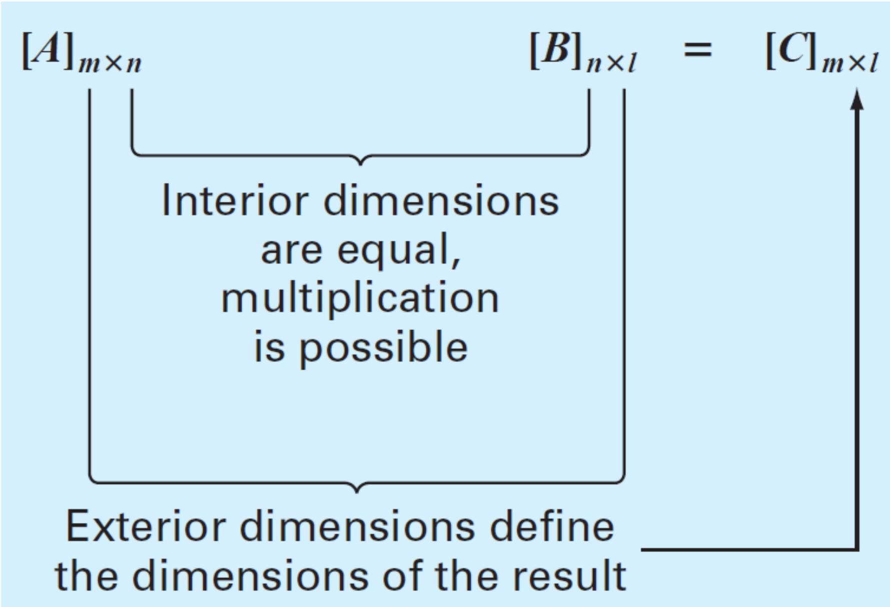
$$\begin{bmatrix} 150 & -100 & 0 \\ -100 & 150 & -50 \\ 0 & -50 & 50 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 588.6 \\ 686.7 \\ 784.8 \end{Bmatrix}$$

**A x = b**

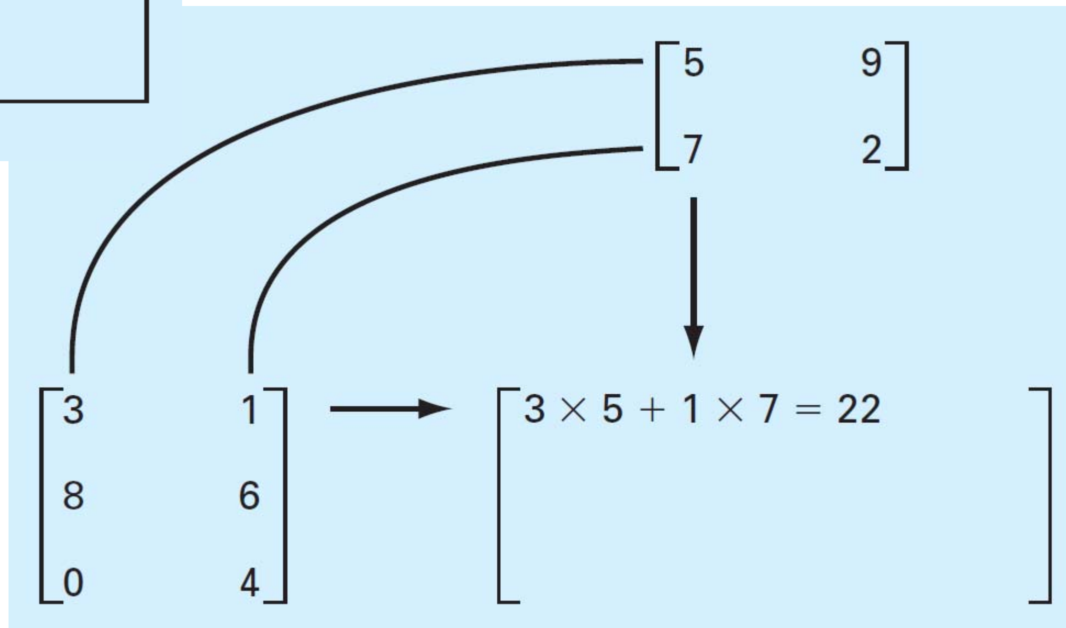
```
A=[150 -100 0; -100 150 -50; 0 -50 50];
b=[588.6; 686.7; 784.8];
x=A\b;              % left division
x=inv(A)*b;         % matrix inverse
xi=[20; 40; 60];    % initial jumpers' positions
xf=xi+x;            % final positions
```



# Matrix multiplication




$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$



# Special matrices

Symmetric

$$[A]=\begin{bmatrix}5 & 1 & 2\\ 1 & 3 & 7\\ 2 & 7 & 8\end{bmatrix}$$

Diagonal

$$[A]=\begin{bmatrix}a_{11} & & \\ & a_{22} & \\ & & a_{33}\end{bmatrix}$$

Identity

$$[A]=\begin{bmatrix}1 & & \\ & 1 & \\ & & 1\end{bmatrix}$$

Upper Triangular

$$[A]=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ & a_{22} & a_{23}\\ & & a_{33}\end{bmatrix}$$

Lower Triangular

$$[A]=\begin{bmatrix}a_{11} & & \\ a_{21} & a_{22} & \\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

Banded

$$[A]=\begin{bmatrix}a_{11} & a_{12} & & \\ a_{21} & a_{22} & a_{23} & \\ & a_{32} & a_{33} & a_{34}\\ & & a_{43} & a_{44}\end{bmatrix}$$

# Reference

- Steven C. Chapra "Applied Numerical Methods with MATLAB", 3rd ed., McGraw Hill, 2012.